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April 2013

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IDA Document D-4811
Log: H 13-000526

INSTITUTE FOR DEFENSE ANALYSES
4850 Mark Center Drive
Alexandria, Virginia 22311-1882

About This Publication

This was conducted by the Institute for Defense Analyses (IDA) under contract DASW01-04-C-0003, Task DE-6-3247, "Comprehensive Assistance to the Strategic Material Security Program (SMSP)," for the Defense Logistics Agency - Strategic Materials. The views, opinions, and findings should not be construed as representing the official position of either the Department of Defense or the sponsoring organization.

Acknowledgments

The authors would like to thank Dr. Arthur Fries, Dr. Peter Picucci, and Dr. Michael Rigdon for reviewing this paper, Ms. Elizabeth Johnson for editing the paper, and Ms. Barbara Varvaglione for helping to prepare this manuscript.

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Executive Summary

Section 14 of the Strategic and Critical Materials Stock Piling Act requires the Department of Defense (DOD) (specifically, Defense Logistics Agency Strategic Materials) to periodically assess the potential for shortfalls of strategic and critical non-fuel materials to occur in the context of a national planning scenario outlined in the Act. The scenario consists of one or more major regional military conflicts followed by a period of military force recovery and regeneration. DOD then recommends to Congress mitigation strategies for materials that could potentially suffer shortfalls during the scenario. The Institute for Defense Analyses (IDA) supports DOD in building and implementing an analytically rigorous process to help the department assess risks arising from potential material shortfalls and set priorities for risk mitigation concerning the materials DOD identifies as being of interest. This document presents a model (the Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM)) IDA has built and used to identify shortfall mitigation strategies that would minimize expected total risk, while satisfying an expected total cost constraint and satisfying constraints on the expected risks arising from possible shortfalls in individual materials.

DOD's analytical process selects materials of interest, estimates the material shortfalls occurring in one or more planning scenarios, assesses shortfall risk, identifies promising shortfall mitigation strategies, and assesses the strategies' relative costs and mitigation effectiveness. This model uses those results and identifies an optimal set of strategies for mitigating the shortfalls. The mitigation strategies considered in the analysis in this document are: (1) Stockpiling: acquisition and storage in the U.S. National Defense Stockpile, (2) Buffer Stocks: acquisition by vendor and storage in vendor-managed buffer stock inventories, (3) Export Guarantee: reduced government guarantees of supplies of material used to produce goods to be exported during the scenario, (4) Substitution: use of substitute materials or goods during the scenario, and (5) Extra Buy: increased U.S. buys of foreign supplies from reliable suppliers during the scenario. Each mitigation strategy acts as an effective source of supply (or reduction in demand) for one or more materials in shortfall. Each strategy has a different capacity (expected supply provided or demand reduced) and a different expected cost for each material. The effectiveness of each strategy in reducing risk depends on how much risk is created by each material shortfall to begin with and how much each strategy reduces each shortfall. The risk created by the shortfalls, the extent to which each strategy can reduce each shortfall, and the cost of each strategy were evaluated in the preparation of the DOD

Strategic and Critical Materials 2013 Report on Stockpile Requirements. The model described here uses those values in solving a linear (or non-linear) programming problem (depending on the form of the risk function—the assumed relationship between shortfall size and shortfall consequences) to identify an optimal set of strategies for mitigating the shortfalls, within cost and risk constraints set by the user.

This document demonstrates the functionality of the OPTIM-SM model by presenting analyses performed on the data developed for the DOD *Strategic and Critical Materials 2013 Report on Stockpile Requirements*. The model is used to identify optimal sets of mitigation strategies for the material shortfalls, with constraints on expected total costs, mitigation strategy capacities, and individual shortfall risks remaining after mitigation. Three initial optimal solution cases are considered for one scenario, in which the maximum fractions of initial shortfall risk for each material allowed to remain after mitigation are 1.00, 0.30, and 0.24. In each case, total mitigation cost is constrained at $50 million and upper bounds on the capacities of the shortfall mitigation strategies are those used in the *2013 Report on Stockpile Requirements*. The results show that as residual risk constraints are tightened (in the second and third cases), the shortfalls of some materials must be reduced below the levels to which they are reduced in the unconstrained, minimum total risk (for the given cost) case (the first case). That, in turn, requires the diversion of resources that had been spent in the first case on reducing risk arising from the shortfalls of other materials. But the further shortfall reductions in the second and third cases cost more (in dollars spent per unit of risk reduced) than the original (unconstrained) reductions. Therefore, because total cost is fixed, total risk must increase. Thus, this demonstration shows how the model accounts for such solution constraints.

In addition to the three optimal solution cases, three experiments are performed to show how the model responds to other changes in input data. First, the constraint on total cost is raised from $50 million to $80 million such that expected total risk is driven to near-zero. Runs varying the maximum remaining fractions of initial shortfall risk, similar to the three cases above, are performed and they demonstrate, as expected, relationships between total risk remaining and constraints on risk remaining from individual materials similar to those shown in the three optimal solution cases. Second, the probability of war—the occurrence of the scenario—is raised significantly. The model shows, as expected, that the higher probability of war raises the expected costs of increased U.S. buys of foreign material supplies at the time of war. This can make buys of foreign material at the time of war unattractive relative to other possible shortfall mitigation strategies. Third, a nonlinear function is adopted so that shortfall consequences will increase nonlinearly with shortfall amount (representing the foregoing of less important applications of a material before more important applications). Results show, as expected, further reduced risk as shortfalls are mitigated below their original values.

Finally, the document discusses a few other observations regarding the model's performance and potential next steps in its development. In sum, the OPTIM-SM model can use information developed in the course of DOD's process for managing strategic and critical materials risk and can integrate many planning parameters to identify material shortfall mitigation options that efficiently balance risks within specific cost constraints. Next steps should increase the model's practical utility to the Department of Defense and potentially other entities engaged in managing risk analogous to that posed by potential shortages of strategic and critical materials.

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Appendices

1. Introduction

Section 14 of the Strategic and Critical Materials Stock Piling Act[1] requires the Department of Defense (DOD) to periodically assess the potential for shortfalls of strategic and critical non-fuel materials in the context of a planning scenario consisting of one or more major regional military conflicts followed by a period of military force recovery and regeneration. DOD then recommends to Congress mitigation strategies for materials that could potentially suffer shortfalls during the scenario. The Institute for Defense Analyses (IDA) supports DOD in building and implementing an analytically rigorous process to help the department assess risks arising from potential material shortfalls and set priorities for risk mitigation concerning the materials DOD identifies as being of interest. This document presents a model IDA has built and used to identify shortfall mitigation strategies that would minimize expected total risk while satisfying an expected total cost constraint and satisfying constraints on the expected risks arising from possible shortfalls in individual materials.

In its *Strategic and Critical Materials 2013 Report on Stockpile Requirements.*[2] DOD estimated defense and essential civilian demand for 76 materials of interest and compared those demands against supplies of the materials that were judged to be available during the planning scenario DOD adopted for this year's report. DOD found shortfalls—supply insufficient to meet demand—for approximately a third (23) of these materials. DOD then evaluated five strategies as options for mitigating each potential material shortfall: (1) Stockpiling: acquisition and storage in the U.S. National Defense Stockpile, (2) Buffer Stock: acquisition by vendor and storage in vendor-managed buffer stock inventories, (3) Export Guarantee: reduced government guarantees of supplies of material used to produce goods to be exported during the scenario, (4) Substitution: use of substitute materials or goods during the scenario, and (5) Extra Buy: increased U.S. buys of foreign supplies from reliable suppliers during the scenario. Based on evaluations of the mitigation measure capacities, probabilities of operating successfully, and expected costs, as they pertain to each material, DOD recommended to Congress which strategies to use, and to what extent, so as to eliminate the shortfalls of each of the 23 materials.

[1] Strategic and Critical Materials Stock Piling Act, 50 USC Section 98h-5 (2013).

[2] Department of Defense, *Strategic and Critical Materials 2013 Report on Stockpile Requirements* (Washington, DC: Under Secretary of Defense for Acquisition, Technology and Logistics, January 2013).

In making those assessments and evaluations, DOD moved beyond the traditional National Defense Stockpile planning process, of estimating material shortfalls and recommending that the shortfall amounts be acquired and stored in the stockpile, to a risk-based process of evaluating stockpiling along with other cost-effective alternatives for mitigating material shortfall risk. The model presented in this document was developed to further advance DOD's material risk management process: to include assessing the most promising set of risk mitigation strategies across materials, within cost and risk constraints. It addresses 19 of the 23 materials (the other four are proprietary materials handled separately).

2. Strategic and Critical Material Risk Assessment and Mitigation Process

To support DOD in identifying potential strategic and critical material shortfalls and assessing options for mitigating them, the IDA team conducted several types of risk assessments and analyses. These drew upon the new analytic structure and techniques that were developed for this purpose, known as the Risk Assessment and Mitigation Framework for Strategic Materials (RAMF-SM). The Demonstration Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM, for Optimization-Strategic Materials) presented in this document was developed as part of RAMF-SM.

The risk assessment and mitigation process consists of the following seven steps that are repeated periodically to allow DOD to manage strategic material risk on a continuous basis:

1. Select materials of interest
2. Estimate material shortfalls in planning scenario
3. Assess shortfall risk
4. Identify promising shortfall mitigation options
5. Assess the options' relative costs and mitigation effectiveness
6. Identify/recommend most promising option set—potentially within a budget constraint
7. Begin cycle again as appropriate

These analytical process steps are described in detail in the *Strategic and Critical Materials 2013 Report on Stockpile Requirements*. These steps are outlined briefly here. In the first step, DOD identifies the materials to evaluate, based on their importance to the department or the U.S. civilian economy and the potential for them to experience shortages in the statutory planning scenario. In the second step, those materials are evaluated using a set of supply and demand models to estimate on a time-phased basis whether the materials would experience shortfalls during the planning scenario and to what extent. These first two steps (and the last) constitute the traditional National Defense Stockpile analytical process that formed the basis for past material stockpiling recommendations.

Steps three through six constitute the new risk-based process for managing potential material shortfalls. In step three, the risks posed by the potential shortfalls are assessed by estimating the probability of the planning scenario and the consequences of each estimated material shortfall. The risk posed by each shortfall (i.e., for each material) is calculated as the product of the scenario probability and a numerical value representing the shortfall consequences. In step four, additional potential shortfall risk mitigation options, beyond stockpiling, are identified for consideration in the risk management process. In the analyses supporting the *2013 Report on Stockpile Requirements*, four more options were identified: buffer stock inventories, reduced guarantees of material supplies for exports, substitution, and increased buys of foreign-supplied materials. In step five, the shortfall risk mitigation options costs and effectiveness are evaluated. In the analyses supporting the *2013 Report*, the options' expected net costs were assessed on a time-phased basis. Their effectiveness was assessed for each material in terms of their capacities to mitigate shortfalls and their probabilities of success in the event they were implemented. In step six, the most promising or cost-effective set of options were identified and recommended for implementation. For the *2013 Report*, mitigation options sufficient to eliminate all shortfalls were selected and the same order of preference for the options was applied to each material: Extra Buy, then Substitution, then Export Guarantees, then Stockpiling. The primary consideration was cost relative to the extent to which each option would mitigate a shortfall. However, increased buys of foreign materials was deemed to be the most preferred option, despite its slightly higher expected cost, because it provides a supply of the material in question without imposing any further limitations on material users.

The OPTIM-SM model presented in this study is to be used in step six to allow the identification of the most promising set of material shortfall risk mitigation options, for a set of shortfalls, within user-selected cost and risk constraints. Identifying optimal mitigation options within a budget constraint is one new analytical capability for DOD that the model provides. The model also newly allows optimal shortfall mitigation options to be identified within constraints placed on residual risk values (resulting from shortfalls potentially partly mitigated) associated with all or with individual material shortfalls. It allows shortfall mitigation options to be fit to tailored cost and risk constraints as desired by the user. This is accomplished by solving a linear or non-linear programming problem, depending on the form of the risk function (specifically the assumed relationship between shortfall size and shortfall consequences).

3. Modeling Objective and Scope

The OPTIM-SM model finds the mix of mitigation options to be used, and to what extent, to minimize expected total risk while satisfying constraints on expected total cost and on expected risks of the individual materials.

The model integrates the following information for all of the materials:

- Probability of scenario resulting in shortfalls
- Length of planning period
- Initial shortfall—units and costs (product of units and material price per unit)
- Shortfall consequences
- Effectiveness and cost parameters of chosen shortfall mitigation measures (five were used for this demonstration; others are possible):
  - Stockpiles
  - Buffer stocks
  - Reduced government guarantees of material supplies for use in exports
  - Substitution
  - Increased U.S. buys of foreign materials
- Probabilities of success of mitigation measures
- Conflict price increases for materials
- Stockpile sale recoupment factor
- Buffer Stock rental cost factor
- Extra Buy optional conflict price factor multiple
- Discount rate for future expenditures

That information was developed for and used in the *2013 Report on Stockpile Requirements* to characterize the statutory planning scenario, to estimate material shortfalls, and to assess the cost-effectiveness of shortfall mitigation options. Thus, the quantities the model uses were obtained or calculated from information that was developed for and used in the *2013 Report*. The model relies on previously-developed

information but it allows calculations to be performed so that optimal choices of shortfall mitigation options can be identified under cost and risk constraints in ways that are not practical without the model when dealing with even modest numbers of materials and options.

This effort, drawing on the *2013 Report on Stockpile Requirements*, models five possible shortfall mitigation strategies. It should be recognized, however, that a mitigation strategy is any activity that can increase material supply and/or decrease material demand, and it is characterized by its cost and by the change it generates in supply and/or demand. Risk, upon the application of any strategy, is a function of still-unsatisfied shortfall. Thus, other shortfall mitigation strategies (e.g., increased material production, material recycling, futures contracts) can also be modeled with this approach, so long as their attributes are characterized in the terms set out here.

4. Measure of Risk

As noted in Chapter 1, in preparing the *2013 Report on Stockpile Requirements* DOD moved beyond the traditional stockpile planning process, of estimating shortfalls and recommending that they be stockpiled, to a risk-based process of assessing material shortfall risk and evaluating stockpiling along with other cost-effective alternatives for mitigating that risk. To do that DOD had to adopt a definition of risk, which this study uses for the optimization model.

The measure of risk is as follows.

Expected Risk = Initial Shortfall Risk × Expected Shortfall Remaining Risk Factor where:

> Initial Shortfall Risk = Probability of Shortfall (Scenario) × Shortfall Consequences (Both are determined by expert elicitation.)
>
> Expected Shortfall Remaining Risk Factor = (Expected Shortfall Remaining / Initial Shortfall) $^{\text{exponent}}$

- Expected Shortfall Remaining is the Initial Shortfall minus the supply increase or demand decrease resulting from the mitigation measures, each of which has a different capacity and effectiveness.
- Initial Shortfall is determined by supply and demand modeling for each material. (Step two of the shortfall risk assessment and mitigation process discussed in Chapter 2.)
- The "exponent," which can be equal to or greater than one, is a factor that is capable of accounting for the effect of shortfall consequences increasing nonlinearly with shortfall amount. (Less important applications of a material would tend to be foregone before more important applications.)

As noted above, the measure of risk used in these analyses is the product of shortfall probability and shortfall consequences. Shortfall probability is ordinarily taken (unless one is modeling an alternative scenario) to be the probability of the statutory planning scenario, which is in turn taken to be the probability of the military conflict(s) represented by the scenario in the planning time frame. Shortfall consequences are ascertained for each material independently based on the size of the shortfall compared to annual demand and the applications of the material in question. In the course of preparing

the *2013 Report on Stockpile Requirements*, experts were provided information on shortfall sizes relative to annual material demands, material applications, and the timeframe of the planning scenario. They evaluated shortfall consequences using their judgment and a ratio scale common to all materials.[3] A different group of experts evaluated shortfall probability using their judgment regarding the probability of the military conflict(s) represented by the planning scenario. These efforts are described in more detail in the *2013 Report*.

After shortfall mitigation measures are applied, risk is equal to Initial Shortfall Risk multiplied by an Expected Shortfall Remaining Risk Factor. As stated, that factor is equal to the fraction of the shortfall remaining, which can be raised to an exponent greater than one to reflect the non-linear increase in shortfall risk with shortfall size that results from less important uses of a material being foregone in a shortage before more important uses. The model is fully capable of solving for optimal shortfall mitigation options with the exponent greater than one, but as is noted later in this document, more data are necessary to ascertain what the exponent should be. For ease of discussion and because the IDA team does not yet have that data, the remainder of this document focuses on solutions with the exponent equal to one.

[3] Shortfall consequences have been assumed to be independent of each other, which has enabled total risk to be calculated by summing individual shortfall risks. This may overstate total risk somewhat, in that shortfall consequences can be redundant and not additive if the shortfalls affect the production of the same goods.

5. Costs of Mitigation Options

The additional shortfall mitigation options now considered in DOD's new process for managing material shortfall risk differ from traditional stockpiling with respect to their effectiveness under different circumstances and their cost. Therefore, mitigation option costs must be estimated and taken into account, along with risk-mitigating effectiveness, in optimizing the choices of mitigation measures for any potential strategic material shortfall-inducing scenario.

The mitigation options are described in detail in the *2013 Report on Stockpile Requirements.* They are summarized briefly here:

- Stockpiling: the government buying and holding a supply of material to be used in the event of a shortage during a conflict or crisis; a stockpile may be sold and its value recouped once it is no longer needed.
- Buffer Stocks: the government contracting with a material supplier or manufacturer to purchase and maintain a specified inventory until needed; the government would pay a rental cost before it was needed and then the acquisition cost at the time it was used.
- Export Guarantees: the government declining to guarantee the availability of materials used to produce some of the goods exported by the United States during the planning scenario.
- Substitution: using alternative materials or alternative goods containing different materials to meet demands during the planning scenario.
- Extra Buy: buying additional materials on the spot market (at potentially higher prices) during the planning scenario.

Formulas for the costs of the mitigation options are as follows.

Stockpiling Net Cost

(Amount Planned × Price per Unit) – (Stockpile Recoupment Factor × Stockpile Recoupment Period Discount Factor × Amount Planned × Price per Unit)

Expected Buffer Stock Cost

(Buffer Stock Rental Cost Factor × Length of Planning Period × Average Discount Factor × Amount Planned × Price per Unit) + (Probability of

War × Average Discount Factor × Probability of Success × Amount Planned × Price per Unit)

Export Guarantees Cost

None

Substitution Cost

None

Expected Extra Buy Cost

Probability of War × Average Discount Factor × Probability of Success × Amount Planned × Price per Unit × Wartime Price Factor × Extra Buy Conflict Price Factor Multiple

Expected net cost formulations are devised for each mitigation measure evaluated by the OPTIM-SM model. Discount factors are applied to all future costs and benefits.[4] The formulations set out here were devised for the *2013 Report on Stockpile Requirements* and their development is discussed there. Costs here are costs to the U.S. government, so the options, Export Guarantees and Substitution, have costs of zero.[5] Net cost is particularly important for stockpiling in that it accounts for recoupment—the sale of a stockpiled material after it is no longer needed to mitigate shortfall risk.[6] Expected cost is particularly important to the Extra Buy option in that the costs of acquiring materials using that option would not be incurred unless the scenario were to occur. As noted below, the capacities of some options to mitigate shortfalls are limited, so even a zero-cost option cannot necessarily be counted upon to completely eliminate any given shortfall.

As an alternative to considering net expected costs when evaluating shortfall mitigation options, the model can also be configured to consider acquisition costs or peacetime budget outlays alone. In such a case, the option cost formulations would be revised to eliminate recoupment for stockpiling and costs incurred during the scenario for the Buffer Stock and Extra Buy options. Such analysis could be useful where budgetary impact was an important determinant of whether shortfall mitigation options would be adopted.

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4 As noted below, the planning period for calculating discount factors for the Buffer Stock and Extra Buy options and Buffer Stock rental costs was taken, from the *2013 Report on Stockpile Requirements*, to be five years.

5 Because the Export Guarantees option is postulated to be only the reduction of government guarantees of material supplies for goods to be exported, the government would incur no cost in implementing it.

6 In the *2013 Report on Stockpile Requirements*, recoupment was assumed to take place after a 20 year planning horizon.

6. Algebraic Description of Optimization-Strategic Materials (OPTIM-SM) Model

This chapter presents the OPTIM-SM model in algebraic terms. Let the appropriate parameters and computed variables be vectors with dimensionality equal to the number of materials.

Define:

Initial Data

IS = initial shortfall (units of material)

IC = initial shortfall cost ($ millions)

IR = initial shortfall risk (determined by expert elicitation)

Expected Risk

X1 = number of units of material planned to be provided by Stockpiling

X2 = number of units of material planned to be provided by Buffer Stocks

X3 = number of units of material planned to be provided by Export Guarantees[7]

X4 = number of units of material planned to be provided by Substitution

X5 = number of units of material planned to be provided by Extra Buy

The variables X1, X2, X3, X4 and X5 are the numbers of units of material that should be planned to be provided by each candidate mitigation option. They are the decision variables solved for by the model.

P1 = probability of success of Stockpiling

P2 = probability of success of Buffer Stocks

P3 = probability of success of Export Guarantees

[7] This option really involves a reduction in material demand that is deemed important enough for the government to provide for rather than an addition of material supply, although mathematically one can and the model does treat it as a source of supply. Similarly, Substitution may be a supply of an alternative material or it may be a reduction in demand caused by the use of functional substitutes but mathematically it may also be treated as a source of supply.

P4 = probability of success of Substitution

P5 = probability of success of Extra Buy

Y1 = P1 X1 = expected number of units of material provided by Stockpiling

Y2 = P2 X2 = expected number of units of material provided by Buffer Stocks

Y3 = P3 X3 = expected number of units of material provided by Export Guarantees

Y4 = P4 X4 = expected number of units of material provided by Substitution

Y5 = P5 X5 = expected number of units of material provided by Extra Buy

TY = Y1 + Y2 + Y3 + Y4 + Y5 = expected number of units of material provided by measures 1 through 5[8]

SR = IS – TY = expected shortfall remaining

SRP = SR/IS = expected shortfall remaining as a fraction of initial shortfall

SRR = SRP $^{\text{exponent}}$ = expected shortfall remaining risk factor

R = SRR IR = expected risk

Expected Shortfall Risk for All Materials

TR = sum of expected risk R over all materials = expected total risk

Expected Cost

PW = probability of war

L = length of planning period

Q = IC/IS = price per unit of material

QF = wartime price factor

QM5 = conflict price factor multiple for measure 5

D = discount rate

RP = Stockpile recoupment factor

LR = length of recoupment period

RDF = recoupment period discount factor = 1 / (1 + D)$^{\text{LR-1}}$

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[8] It is assumed, here and in the *2013 Report on Stockpile Requirements*, that there is no interaction between the different types of demand mitigation (e.g., substitution and export guarantees) and thus their effects are additive.

BRF = Buffer Stock rental cost factor

ADF = average discount factor = $(1 / L) ((1+D)^0 + \ldots + (1 + D)^{L-1})$

C1 = Stockpiling Cost = X1 Q – RP RDF X1 Q

C2 = Expected Buffer Stock Cost = BRF L ADF X2 Q + P2 PW ADF X2 Q

C3 = Export Guarantees Cost = 0

C4 = Substitution Cost = 0

C5 = Expected Extra Buy Cost = P5 PW ADF X5 Q QF QM5

C = C1 + C2 + C3 + C4 + C5 = expected cost of mitigation measures 1 through 5

Expected Cost for All Materials

TC = sum of expected cost C over all materials = expected total cost

Optimization

The optimization model is as follows.

Choose X1, X2, X3, X4, X5 over all materials to minimize TR (expected total risk) subject to:

SR $\geq$ 0, for all materials[9]

TC $\leq$ upper bound on expected total cost

R $\leq$ upper bounds on individual expected risks, for all materials

X3, X4 and X5 $\leq$ upper bounds on Export Guarantees, Substitution and Extra Buy, for all materials

The foregoing equations define the working of the OPTIM-SM model. The initial conditions are the material shortfalls, the shortfall costs, and the initial (unmitigated) shortfall risks. The decision variables solved for by the model are the numbers of units of material that should be planned to be provided by each candidate mitigation option. The model estimates the amounts by which each mitigation option would be expected to reduce each shortfall and thus the shortfalls and risks expected after mitigation.[10] It estimates the expected costs of each mitigation option for each material. It calculates expected risk remaining and expected cost for each option over all materials. It optimizes for the set of mitigation options that minimizes total risk, subject to possible bounds on:

[9] Expected shortfalls cannot be negative.

[10] It should be noted that while the variables are described here as probabilities of success for the mitigation options, for some options (like Substitution) they are best interpreted as fractions of the planned amounts that will be provided (as opposed to probabilities). Because the model uses expected amounts of material obtained, the choice does not matter in mathematical terms.

the capacities of the mitigation options, total cost, and remaining risk for each shortfall material. Mitigation option capacities may be bounded by real world implementation challenges or the availability of materials or substitute materials or goods. Total cost and remaining risk for each shortfall are bounded as chosen by the model user.[11] Solutions may be obtained non-linearly if the optional exponent to reflect the non-linear increase in shortfall risk with shortfall size is used or linearly if it is not. Finally, it should be noted that some formulated problems may have no feasible solution (for example, problems with both low cost and low risk bounds). If such problems are discovered, the range of allowable solutions must be expanded until feasible solutions can be found.

[11] As discussed after Table 7 below, a user might impose bounds on remaining risk for individual materials to reduce the likelihood that a shortfall that is costly to mitigate would end up preventing the production of important goods during a crisis scenario.

7. Running the Model

The OPTIM-SM model is implemented in Microsoft Excel (2007) on a personal computer. There are 95 decision variables (given the 19 shortfall materials evaluated in this example). There are about 300 input parameters. The model can be solved in a few seconds.

The software used to solve the optimization problems is Premium Solver Pro, an add-on to the standard Microsoft Excel package. It is provided by Frontline Systems, Inc., the company that provides the optimization software Solver to Microsoft for inclusion in the standard Microsoft Excel package. The reason that Premium Solver Pro is used for this model is that it solves nonlinear programming problems, which is required for the nonlinear version. If a user were satisfied with the linear version the standard Microsoft Excel Solver could be used. It handles linear programs with up to 200 decision variables.

Chapters 8 to 13 will present data regarding the materials, material shortfalls, and mitigation options, and the planning assumptions and parameters developed and used in the *2013 Report on Stockpile Requirements*. They will demonstrate how the model works to identify optimal sets of mitigation options for the shortfalls listed in the *2013 Report* under several different sets of cost and remaining risk bounds for each shortfall material.

8. Material and Shortfall Data

Data for initial shortfalls, initial shortfall risks, prices, conflict price factors, and the costs of the initial shortfalls from the *2013 Report on Stockpile Requirements* are as follows (note that cost of initial shortfall = initial shortfall (units) × current price).

Table 1. Material and Shortfall Data

Material	Units	Initial Shortfall (Units)	Initial Shortfall Risk	Current Price ($ per Unit)	Wartime Price Factor	Cost of Initial Shortfall ($)
Aluminum Oxide	short tons	231,485	0.043	569	2.94	131,669,580
Antimony	short tons	22,575	0.041	8,063	1.83	182,035,148
Beryllium Metal	short tons	52	0.071	310,017	4.22	16,120,867
Bismuth	1000 pounds	3,630	0.040	11	2.47	39,585,361
Chromium Metal	short tons	718	0.015	14,880	2.07	10,684,523
Dysprosium	MT Oxide	47	0.025	460,087	17.27	21,644,206
Erbium	MT Oxide	124	0.047	100,454	19.93	12,428,255
Fluorspar Acid Grade	short tons	56,322	0.012	383	1.22	21,544,210
Gallium	kilograms	17,686	0.040	593	1.93	10,479,721
Germanium	kilograms	28,888	0.044	1,234	4.04	35,661,041
Manganese Metal – Electrolytic	short tons	7,406	0.025	3,100	1.05	22,956,120
Scandium	KG Oxide	572	0.026	1,350	132.59	772,298
Silicon Carbide	short tons	81,869	0.029	1,147	1.42	93,877,495
Tantalum	1000 pounds	623	0.030	68	7.39	42,071,357
Terbium	MT Oxide	7	0.022	996,933	13.03	7,160,960
Thulium	MT Oxide	20	0.040	165,000	158.17	3,313,529
Tin	Metric tons	19,428	0.029	21,417	1.48	416,090,282
Tungsten	1000 pounds	11,289	0.034	7	5.82	84,261,684
Yttrium	MT Oxide	1,899	0.050	44,850	16.41	85,174,115
Total			**0.663**			**$1.238B**

9. Planning Assumptions and Parameters

Planning assumptions and parameters are as follows.

- Probability of war – 0.0037
- Length of planning period – five years
- Stockpile recoupment factor – 0.84
- Buffer Stock rental cost factor – 15% per year
- Extra Buy conflict price factor multiple – 1.0
- Discount factor for five-year planning period assuming event occurs at mid-period – 0.992
- Discount factor for stockpiling assuming recoupment after 20 years – 0.927
- SRP exponent = 1.0.

These assumptions and parameters were developed for the *2013 Report on Stockpile Requirements*.

10. Probabilities of Success for Mitigation Options

Probabilities of success (in percent given an attempted use) for the five mitigation options, developed for the *2013 Report on Stockpile Requirements*, are as follows. Values of zero indicate that the option is not feasible for that material.

Table 2. Mitigation Option Probabilities of Success (%)

Material	Stockpile	Buffer Stock	Export Guarantees	Substitution	Extra Buy
Aluminum Oxide	77	67	46	69	73
Antimony	75	67	41	59	59
Beryllium Metal	76	67	0	0	0
Bismuth	76	70	45	61	61
Chromium Metal	83	70	46	59	0
Dysprosium	74	43	39	45	0
Erbium	70	43	41	59	0
Fluorspar Acid Grade	77	71	44	66	75
Gallium	77	64	41	55	67
Germanium	78	67	44	59	64
Manganese Metal – Electrolytic	75	64	45	0	74
Scandium	74	44	37	88	0
Silicon Carbide	76	67	51	64	71
Tantalum	77	65	41	0	0
Terbium	77	42	40	49	0
Thulium	70	42	40	59	0
Tin	73	67	44	67	78
Tungsten	77	65	46	57	62
Yttrium	71	43	35	48	54

11. Upper Bounds on Export Guarantees, Substitution, and Extra Buy

Upper bound capacities relative to each material shortfall for Export Guarantees, Substitution, and Extra Buy are as follows (multiplied by the appropriate shortfall amounts they form the upper bounds on variables X3, X4, and X5). These capacities were developed for the *2013 Report on Stockpile Requirements*.

Table 3. Mitigation Option Capacities (Relative to Shortfall Amounts)

Material	Units	Export Guarantees	Substitution	Extra Buy
Aluminum Oxide	short tons	0.26	1.35	0.11
Antimony	short tons	0.26	.29	0.33
Beryllium Metal	short tons	0	0	0
Bismuth	1000 pounds	0.36	0.04	0.44
Chromium Metal	short tons	4.62	1.66	0
Dysprosium	MT Oxide	0.98	0.43	0
Erbium	MT Oxide	0.23	0.19	0
Fluorspar Acid Grade	short tons	3.10	15.13	3.27
Gallium	kilograms	0.74	0.59	0.22
Germanium	kilograms	0.31	0.27	0.25
Manganese Metal – Electrolytic	short tons	0.49	0	1.82
Scandium	KG Oxide	0.11	1.10	0
Silicon Carbide	short tons	0.40	0.80	0.23
Tantalum	1000 pounds	1.31	0	0
Terbium	MT Oxide	1.67	1.67	0
Thulium	MT Oxide	0.30	0.60	0
Tin	Metric tons	0.77	0.25	1.13
Tungsten	1000 pounds	0.44	0.29	0.62
Yttrium	MT Oxide	0.24	0.24	0.06

12. Modeling Results—Example Optimizations

The IDA team used the OPTIM-SM model with the data set forth in Chapter 8 to identify optimal sets of mitigation options for the material shortfalls. Constraints on expected total costs, on the use of the Export Guarantees, Substitution, and Extra Buy options, and on individual shortfall risks remaining after mitigation, were as described below. The modeling results are then summarized and discussed on the following pages.

Constraints:

- The upper bound on expected total cost is $50 million.
- The upper bounds on the Export Guarantees and Substitution options are as given above.
- The use of Extra Buy is constrained to be equal to its upper bounds, except for Fluorspar Acid Grade and Manganese Metal – Electrolytic, for which its use is set at (1/probability of success) × Initial shortfall.[12]

Definition of Three Cases:

- The upper bound multiples for individual risks (i.e., the fractions of initial shortfall risk for each material allowed to remain after mitigation) are 1.00, 0.30 and 0.24 of initial risks.[13] As discussed after Table 7 below, reducing these bounds forces shortfalls to be reduced more evenly, which might be desirable as a way to prevent unmitigated shortfalls from preventing the production of critical goods during a conflict.

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[12] This constraint makes Extra Buy the preferred option for mitigating shortfalls, consistent with the recommendation made in the *2013 Report on Stockpile Requirements*, even though its expected costs are higher than those of Substitution and Export Guarantees. Note that where the probability of success is less than 1.0, the number of units of material planned to be provided by Extra Buy (X5) will be greater than the shortfall amount (IS) but the expected number of units of material provided by Extra Buy (Y5) will be equal to the shortfall amount.

[13] Where the SRP exponent is equal to 1 (linear model), these are also the upper bounds on the fractions of shortfall remaining (SRP for each material).

- Note: Initial risks (in Table 1 above) are multiplied by 100 for ease of interpretation of results (thus initial total risk = 66.3 for each of these cases on this scale).[14]

Results for Three Cases:

- For upper bound multiples for individual risks = 1.00: Expected total risk = 4.0; No expected risk for 15 materials
- For upper bound multiples for individual risks = 0.30: Expected total risk = 6.2; No expected risk for 12 materials
- For upper bound multiples for individual risks = 0.24: Expected total risk = 10.8; No expected risk for eight materials

Allocations of expenditures to stockpiles of various materials change significantly as upper bounds are reduced. For instance, as can be seen by comparing Tables 4, 6, and 8, going from upper bounds on individual shortfall risks of 1.00 to 0.30 to 0.24 requires more stockpiling of relatively expensive materials, increases remaining risks on some of the other materials, and increases expected total risk. This is explained further following Table 7 below.

[14] See Material and Shortfall Data.

A. Optimal Solution Case 1 (1.00): Expected Costs and Risks and Number of Units

1. Expected Costs of Optimal Solution, Upper Bound Multiples for Individual Risks = 1.00

Expected costs of the optimal (minimum total remaining risk) solution for upper bound multiples for individual shortfall risks of 1.00 are as follows.

Table 4. Expected Costs for Upper Bound Multiples for Individual Risks = 1.00

Material	Stockpile Costs ($ million)	Buffer Stock Costs ($)	Export Guarantees Costs ($)	Substitution Costs ($)	Extra Buy Costs ($ thousand)
Aluminum Oxide	0	0	0	0	114
Antimony	0	0	0	0	238
Beryllium Metal	4.7	0	0	0	0
Bismuth	6.3	0	0	0	96
Chromium Metal	0	0	0	0	0
Dysprosium	2.7	0	0	0	0
Erbium	3.1	0	0	0	0
Fluorspar Acid Grade	0	0	0	0	96
Gallium	.7	0	0	0	11
Germanium	5.5	0	0	0	85
Manganese Metal – Electrolytic	0	0	0	0	88
Scandium	0	0	0	0	0
Silicon Carbide	0	0	0	0	80
Tantalum	5.6	0	0	0	0
Terbium	0	0	0	0	0
Thulium	.6	0	0	0	0
Tin	0	0	0	0	1,998
Tungsten	0	0	0	0	692
Yttrium	17.2	0	0	0	166
Total	**$46.3M**				**$3.7M**

2. Expected Risks of Optimal Solution, Upper Bounds on Individual Risks = 1.00

Expected risks of the optimal solution for upper bound multiples for individual risks of 1.00 are as follows. Note that risks are zero for 15 of the 19 materials. The expected total risk is 4.0.

Table 5. Expected Risks for Upper Bound Multiples for Individual Risks = 1.00

Material	Units	Expected Shortfall Remaining (Units)	Expected Risk (× 100 scale)	Upper Bound on Expected Risk
Aluminum Oxide	short tons	0	0	4.30
Antimony	short tons	11,910	2.16	4.10
Beryllium Metal	short tons	0	0	7.10
Bismuth	1000 pounds	0	0	4.00
Chromium Metal	short tons	0	0	1.50
Dysprosium	MT Oxide	0	0	2.50
Erbium	MT Oxide	0	0	4.70
Fluorspar Acid Grade	short tons	0	0	1.20
Gallium	Kilograms	0	0	4.00
Germanium	Kilograms	0	0	4.40
Manganese Metal – Electrolytic	short tons	0	0	2.50
Scandium	KG Oxide	0	0	2.60
Silicon Carbide	short tons	9,882	0.35	2.90
Tantalum	1000 pounds	0	0	3.00
Terbium	MT Oxide	0	0	2.20
Thulium	MT Oxide	0	0	4.00
Tin	Metric tons	0	0	2.90
Tungsten	1000 pounds	2,798	0.84	3.40
Yttrium	MT Oxide	231	0.61	5.00
Total			**4.0**	**66.3**

3. Number of Units Expected to be Provided in Optimal Solution, Upper Bounds on Individual Risks = 1.00

The number of units of each material that would be planned to be provided under the optimal solution of the $50 million case with upper bound multiples of individual material risks = 1.00 is set forth in Appendix A. Also set forth there is the number of units of each material that would be planned to be provided under the optimal solution of the material shortfall problem with no upper bounds on individual material risks or cost. (The latter is equivalent to the solution of the problem developed manually for the *2013 Report on Stockpile Requirements*.) It can be seen from these results that the shortfalls, which total over $1.2 billion in cost, can be nearly eliminated for $50 million net cost. While perhaps surprising, this is driven by the availability of low- and no-cost options for mitigating risk (Extra Buy, Substitution, and Export Guarantees) and the lower net cost of stockpiling when the longer-term potential for recoupment is considered.

B. Optimal Solution Case 2 (0.30): Expected Costs and Risks

1. Expected Costs of Optimal Solution, Upper Bound Multiples for Individual Risks = 0.30

Expected costs of the optimal solution for upper bound multiples for individual risks of 0.30 are as follows.

Table 6. Expected Costs for Upper Bound Multiples for Individual Risks = 0.30

Material	Stockpile Costs ($ million)	Buffer Stock Costs ($)	Export Guarantees Costs ($)	Substitution Costs ($)	Extra Buy Costs ($ thousand)
Aluminum Oxide	0	0	0	0	114
Antimony	12.2	0	0	0	238
Beryllium Metal	4.7	0	0	0	0
Bismuth	2.8	0	0	0	96
Chromium Metal	0	0	0	0	0
Dysprosium	2.3	0	0	0	0
Erbium	3.1	0	0	0	0
Fluorspar Acid Grade	0	0	0	0	96
Gallium	0.7	0	0	0	11
Germanium	5.5	0	0	0	85
Manganese Metal – Electrolytic	0	0	0	0	88
Scandium	0	0	0	0	0
Silicon Carbide	0	0	0	0	80
Tantalum	2.0	0	0	0	0
Terbium	0	0	0	0	0
Thulium	0.6	0	0	0	0
Tin	0	0	0	0	1,998
Tungsten	0	0	0	0	692
Yttrium	12.4	0	0	0	166
Total	**$46.3M**				**$3.7M**

2. Expected Risks of Optimal Solution, Upper Bounds on Individual Risks = 0.30

Expected risks of the optimal solution for upper bound multiples for individual risks of 0.30 are as follows. Note that risks are zero for 9 of the 19 materials. The expected total risk is 6.2.

Table 7. Expected Risks for Upper Bound Multiples for Individual Risks = 0.30

Material	Units	Expected Shortfall Remaining (Units)	Expected Risk (× 100 scale)	Upper Bound on Expected Risk
Aluminum Oxide	short tons	0	0	1.29
Antimony	short tons	6,772	1.23	1.23
Beryllium Metal	short tons	0	0	2.13
Bismuth	1000 pounds	1,089	1.20	1.20
Chromium Metal	short tons	0	0	0.45
Dysprosium	MT Oxide	3	0.16	0.75
Erbium	MT Oxide	0	0	1.41
Fluorspar Acid Grade	short tons	0	0	0.36
Gallium	kilograms	0	0	1.20
Germanium	kilograms	0	0	1.32
Manganese Metal – Electrolytic	short tons		0	0.75
Scandium	KG Oxide	0	0	0.78
Silicon Carbide	short tons	9,882	0.35	0.87
Tantalum	1000 pounds	187	0.90	0.90
Terbium	MT Oxide	0	0	0.66
Thulium	MT Oxide	0	0	1.20
Tin	Metric tons	0	0	0.87
Tungsten	1000 pounds	2,798	0.84	1.02
Yttrium	MT Oxide	570	1.50	1.50
Total			**6.2**	**19.9**

This result, when compared to Table 5, shows how, when the optimal solution is constrained, for a given cost, total remaining risk increases. Here, when expected risk constraints for each shortfall material were reduced from 1.00 (no constraints) to 0.30 (maximum expected risk for each material no greater than 30% of initial risk), total expected risk increased from 4.0 to 6.2 (out of 66.3 initially). The tighter risk constraints require some shortfalls to be reduced below the levels to which they are reduced in the unconstrained, minimum total risk (for the given budget) case. Those further reductions require the diversion of resources that had been spent on reducing risk arising from other

shortfalls. But the further reductions cost more (in dollars spent per unit of risk reduced) than the original (unconstrained) reductions. Thus, because the budget is fixed, total risk must go up. All of this is a response that one would expect but it shows how the model accounts for such solution constraints.

In practice one might impose those kinds of constraints out of concern that if some shortfalls were left unmitigated, out of a desire to pursue the most cost-effective overall shortfall solution, those shortfalls might prevent certain industries from producing important goods. Constraining expected risk for all materials forces all or at least most shortfalls to be reduced more evenly, which reduces the likelihood that a shortfall that might be more costly to mitigate would end up preventing the production of important goods during a crisis scenario.

C. Optimal Solution Case 3 (0.24): Expected Costs and Risks

1. Expected Costs of Optimal Solution, Upper Bound Multiples for Individual Risks = 0.24

Expected costs of the optimal solution for upper bound multiples for individual risks of 0.24 are as follows.

Table 8. Expected Costs for Upper Bound Multiples for Individual Risks = 0.24

Material	Stockpile Costs ($ million)	Buffer Stock Costs ($)	Export Guarantees Costs ($)	Substitution Costs ($)	Extra Buy Costs ($ thousand)
Aluminum Oxide	0	0	0	0	114
Antimony	15.5	0	0	0	238
Beryllium Metal	3.6	0	0	0	0
Bismuth	3.5	0	0	0	96
Chromium Metal	0	0	0	0	0
Dysprosium	1.2	0	0	0	0
Erbium	2.2	0	0	0	0
Fluorspar Acid Grade	0	0	0	0	96
Gallium	0	0	0	0	11
Germanium	3.1	0	0	0	85
Manganese Metal – Electrolytic	0	0	0	0	88
Scandium	0	0	0	0	0
Silicon Carbide	0	0	0	0	80
Tantalum	2.7	0	0	0	0
Terbium	0	0	0	0	0
Thulium	.6	0	0	0	0
Tin	0	0	0	0	1,998
Tungsten	.2	0	0	0	692
Yttrium	14.0	0	0	0	166
Total	**$46.3M**				**$3.7M**

2. Expected Risks of Optimal Solution, Upper Bound Multiples for Individual Risks = 0.24

Expected risks of the optimal solution for upper bound multiples for individual risks of 0.24 are as follows. Note that risks are zero for five of the 19 materials. The expected total risk is 10.8.

Table 9. Expected Risks for Upper Bound Multiples for Individual Risks = 0.24

Material	Units	Expected Shortfall Remaining (Units)	Expected Risk (× 100 scale)	Upper Bound on Expected Risk
Aluminum Oxide	short tons	0	0	1.03
Antimony	short tons	5,418	0.98	0.98
Beryllium Metal	short tons	12	1.70	1.70
Bismuth	1000 pounds	871	0.96	0.96
Chromium Metal	short tons	0	0	0.36
Dysprosium	MT Oxide	11	0.60	0.60
Erbium	MT Oxide	30	1.13	1.13
Fluorspar Acid Grade	short tons	0	0	0.29
Gallium	kilograms	3,974	0.90	0.96
Germanium	kilograms	6,933	1.06	1.06
Manganese Metal – Electrolytic	short tons	0	0	0.60
Scandium	KG Oxide	0	0	0.62
Silicon Carbide	short tons	19,649	0.70	0.70
Tantalum	1000 pounds	150	0.72	0.72
Terbium	MT Oxide	0	0	0.53
Thulium	MT Oxide	0	0	0.96
Tin	Metric tons	0	0	0.70
Tungsten	1000 pounds	2,709	0.82	0.82
Yttrium	MT Oxide	456	1.20	1.20
Total			**10.8**	**15.9**

This result reinforces the point made by the previous model run. As expected, when the solution set is constrained further, some shortfalls are reduced further, some increase, and total expected risk increases.

The results in Tables 4–9 show a few patterns. First, the costs of Buffer Stock, Export Guarantees, and Substitution are always zero. This is because the use of the Export Guarantees and Substitution options imposes no costs on the government and the

Buffer Stock option is never used—it is always suboptimal relative to Stockpiling. Second, the Extra Buy costs always sum to \$3.7 million because Extra Buy has been chosen as the first option by policy and the shortfall amounts are always the same in these cases. Thus, the Stockpiling costs always sum to \$46.3—the amount remaining under the cost limit of \$50 million. However, the Stockpiling quantities and costs for individual materials change as the risk constraints force the model away from the least cost set of options.

13. Three Experiments

Following are three experiments and their results. The purpose is to show the OPTIM-SM model response to further changes in important input data.

A. First Experiment—Increase the Budget

Raise expected total cost from $50 million to $80 million and drive the expected total risk to near-zero.

Results:

- For upper bound multiples for individual risks of 1.00 the expected total risk is 0.8 (lowest possible total risk).
- For upper bound multiples for individual risks of 0.20 the expected total risk is 0.9.
- For upper bound multiples for individual risks of 0.07 the expected total risk is 1.9 (lowest possible balanced total risk).

This compares with the previous result for $50 million of the lowest possible expected total risk of 4.0 and the lowest possible balanced expected total risk of 10.8 (achieved with upper bounds on individual risks of 0.24). Furthermore, as expected, the tradeoff between cost and risk with increasingly constrained solution sets is the same as in the $50 million cases.

B. Second Experiment—Increase the Probability of War

Change the probability of war from 0.0037 to 0.1 and explore different rules for Extra Buy.

Results:

- With expected total cost of $120 million, upper bound multiples for individual risks of 0.50, and equality constraints on Extra Buy the expected total risk is 11.4 and the amount spent on Extra Buy is $99 million.
- When there are upper bounds rather than equality constraints on Extra Buy the expected total risk is reduced from 11.4 to 1.1 and the amount spent on Extra Buy is $21 million. Thus, the cost of Extra Buy relative to Substitution and

Reduced Exports goes up and so does the expected cost of making Extra Buy the preferred mitigation option over those no cost options. This follows from the nature of the Extra Buy option—the material needed to cover the shortfall is not purchased unless the conflict occurs.

C. Third Experiment—Adopt a Nonlinear Risk Function

Adopt a nonlinear risk function with exponent 1.5. Recall that $R = IR \times SRR$ and that SRR = (SR / IS) raised to an exponent that is taken here to be 1.5. Examples of this effect on SRR are:

- $0.9^{1.5} = 0.85$
- $0.8^{1.5} = 0.71$
- $0.5^{1.5} = 0.36$

This produces the following results for an expected total cost of $50 million:

- For upper bound multiples for individual risks of 1.00, expected total risk is 3.1 (lowest possible total risk).
- For upper bound multiples for individual risks of 0.15, expected total risk is 4.9.
- For upper bound multiples for individual risks of 0.12, expected total risk is 5.0 (lowest possible balanced total risk).

This compares with the previous results for the expected total cost of $50 million, where the lowest possible total risk of 4.0 and the lowest possible balanced total risk of 10.8 was achieved with the higher upper bounds on individual risks of 0.24. This is to be expected when it is considered that the non-linear relationship between expected shortfall remaining and expected shortfall risk reduces the expected shortfall risk below what it would be under the linear relationship. As noted, this effect is intended to represent the ability of material users to reduce the consequences of a less than complete shortage of a material by foregoing less important uses before foregoing more important uses. The challenge in using the non-linear function for real analyses will be to establish an appropriate value for the exponent based on the relative importance of the applications of the materials that might be foregone in a shortage.

14. Observations and Next Steps

A. Other Model Sensitivities

Solutions are sensitive to all of the planning parameters and technical parameters, as illustrated throughout this document.

Buffer Stocks have not appeared in the cases explored in this study. Under the conditions evaluated in the study examples, Buffer Stocks are not cost-effective relative to the other available mitigation options. This is driven by the relatively high Buffer Stock Rental Factor. Simultaneously decreasing the Stockpile recoupment factor and the Buffer Stock Rental Factor can shift the optimal strategy from Stockpile to Buffer Stock.

B. Concluding Observation

The OPTIM-SM model can use information developed in the course of DOD's process for managing strategic and critical materials risk and can integrate many planning parameters to identify material shortfall mitigation options that efficiently balance risks within a specified budget. The model can allow calculations to be performed so that optimal choices of options can be identified under cost and risk constraints in ways that are not otherwise practical when dealing with even modest numbers of materials and options.

C. Next Steps

Now that the OPTIM-SM modeling capability described here has been developed, the next steps should be those to increase its practical utility to the Department of Defense and potentially other entities engaged in managing risk analogous to that posed by potential shortages of strategic and critical materials. Such steps could be to refine the representation of the economy implicit in the relationships between risk and possible shortfalls and mitigation measures. The time phasing (for example on a year-by-year basis) of shortfalls, the effects of mitigation measures, and costs could be explicitly represented in the model. Defense and civilian shortfalls, with different risks and possibly different mitigation measures, could be represented separately. Estimates of shortfall consequences and the consequences of multiple shortfalls at once could be improved by explicitly (rather than implicitly) considering the applications and industries that require the materials and the importance of those applications and industries. More shortfall mitigation options, like futures contracts, could be added to the existing mix. Market

responses to supply and demand shocks could be modeled more explicitly as could uncertainty and the distribution of possible shortfall and mitigation outcomes.

Another significant step would be to extend the model's representation of raw material supplies and treatment of supply-related risk to downstream processed materials and manufactured goods. With the global supply chains that exist today, it is important for the United States to do more than manage raw material-related risk. If the United States is to assure the functioning of the civilian economy and the defense industrial base, we also need to manage risks to the downstream elements of their supply chains that could threaten the availability of critical goods in the event of a conflict or crisis. The approach taken here could be extended downstream by representing the nodes in the supply chains that are necessary to the output of important industries, assessing potential risks to them, and evaluating potential options for mitigating those risks. Total risk and the cost effectiveness of risk mitigation options for each material would be evaluated on the basis of the ability of the supply chain from beginning to end to deliver important goods to U.S. users, both civilian and military. The model would allow optimal risk mitigation strategies to be identified under various cost and risk constraints.

In addition to the foregoing, the model could be adapted for use outside of the strategic and critical materials context to help identify optimal options, under cost and risk constraints, for mitigating national security risks. Security risks could be modeled as threat scenarios analogous to material shortfalls. Risk mitigation options, like force structure elements or security-related programs, could be modeled analogously to materials risk mitigation options like stockpiling or maintaining buffer stocks. Clearly, significant work would have to be done to develop the data characterizing the baseline risks and the costs and effectiveness of the security options. But the structure of the model would be adaptable to this purpose if the necessary input data were developed.

Appendix A
Shortfall Mitigation Requirements for Upper Bound Multiples for Individual Risks = 1.00

Table A-1 (extracted from DOD's *Strategic and Critical Materials 2013 Report on Stockpile Requirements*) shows the mitigation measures needed to be implemented to avoid shortages in the context of no budget or risk constraints.

Table A-2 shows the model prediction for the measures needed for an expected cost constraint of $50 million where stockpile recoupment can be employed to reduce net costs. The results indicate that the total units of mitigation affordable for Antimony, Silicon Carbide, Tungsten, and Yttrium are insufficient to ameliorate a shortage of those materials.

Units Planned to be Provided, No Upper Bounds on Individual Risks or on Total Cost

Table A-1 gives the number of units planned to be provided in the optimal solution for the case where there are no upper bounds on risks for individual materials or on total cost (and no stockpile recoupment occurs). This is equivalent to the solution developed for the *2013 Report on Stockpile Requirements.*

Table A-1. Units of Material Planned to be Provided with No Upper Bounds on Individual Risks or on Total Cost

Material	Units	Initial Short-fall (Units)	Stock-pile (Units)	Buffer Stock (Units)	Export Guaran-tees (Units)	Substi-tution (Units)	Extra Buy (Units)	Total Mitiga-tion (Units)
Aluminum Oxide	short tons	231,485	0	0	0	308,546	25,463	334,009
Antimony	short tons	22,575	0	0	5,869	6,547	7,450	19,866
Beryllium Metal	short tons	52	68	0	0	0	0	68
Bismuth	1000 pounds	3,630	2,604	0	1,307	145	1,597	5,653
Chromium Metal	short tons	718	0	0	32	1,192	0	1,224
Dysprosium	MT Oxide	47	27	0	46	20	0	93
Erbium	MT Oxide	124	140	0	28	24	0	192
Fluorspar Acid Grade	short tons	56,322	0	0	0	22	75,076	75,098
Gallium	kilograms	17,686	5,161	0	13.088	10,435	3,891	19,500
Germanium	kilograms	28,888	20,159	0	8,955	7,800	7,222	44,136
Manganese Metal – Electrolytic	short tons	7,406	0	0	1	0	10,007	10,008
Scandium	KG Oxide	572	0	0	49	629	0	678
Silicon Carbide	short tons	81,869	0	0	32,748	65,495	18,830	117,073
Tantalum	1000 pounds	623	375	0	817	0	0	1,192
Terbium	MT Oxide	7	0	0	3	12	0	15
Thulium	MT Oxide	20	15	0	6	12	0	33
Tin	metric tons	19,428	0	0	0	3,371	22,012	25,383
Tungsten	1000 pounds	11,288	0	0	4,967	3,274	6,999	15,240
Yttrium	MT Oxide	1,899	1,729	0	456	456	114	2,755

Number of Units Expected to be Provided, Upper Bound Multiples for Individual Risks = 1.00

Table A-2 gives the number of units expected to be provided in the optimal solution, with the given the probabilities of success, for the case where upper bound multiples on risks for individual materials are 1.00 and the upper bound on expected total cost is $50 million (assuming stockpile recoupment occurs).

Note that the total mitigation is less than the initial shortfall for four materials—Antimony, Silicon Carbide, Tungsten and Yttrium. These are denoted by asterisks.

Table A-2. Units of Material Planned to be Provided with Upper Bound Multiples for Individual Risks = 1.00

Material	Units	Initial Short-fall (Units)	Stock-pile (Units)	Buffer Stock (Units)	Export Guaran-tees (Units)	Substi-tution (Units)	Extra Buy (Units)	Total Mitiga-tion (Units)
Aluminum Oxide	short tons	231,485	0	0	0	212,896	18,588	231,485
Antimony	short tons	22,575	0	0	2,406	3,863	4,395	10,664*
Beryllium Metal	short tons	52	52	0	0	0	0	52
Bismuth	1000 pounds	3,630	1,979	0	588	89	974	3,630
Chromium Metal	short tons	718	0	0	15	703	0	718
Dysprosium	MT Oxide	47	20	0	18	9	0	47
Erbium	MT Oxide	124	98	0	12	14	0	124
Fluorspar Acid Grade	short tons	56,322	0	0	0	15	56,307	56,322
Gallium	kilograms	17,686	3,974	0	5,366	5,739	2,607	17,686
Germanium	kilograms	28,888	15,724	0	3,940	4,602	4,622	28,888
Manganese Metal – Electrolytic	short tons	7,406	0	0	0	0	7,405	7,406
Scandium	KG Oxide	572	0	0	18	554	0	572
Silicon Carbide	short tons	81,869	0	0	16,701	41,917	13,369	71,987*
Tantalum	1000 pounds	623	289	0	335	0	0	623
Terbium	MT Oxide	7	0	0	1	6	0	7
Thulium	MT Oxide	20	11	0	2	7	0	20
Tin	metric tons	19,428	0	0	0	2,259	17,169	19,428
Tungsten	1000 pounds	11,288	0	0	2,285	1,866	4,339	8,490*
Yttrium	MT Oxide	1,899	1,228	0	160	219	62	1,668*

Appendix B
Illustrations

Tables

Appendix C
References

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Appendix D
Abbreviations

DOD	Department of Defense
IDA	Institute for Defense Analyses
NDS	National Defense Stockpile
OPTIM-SM	Optimization-Strategic Materials
RAMF-SM	Risk Assessment and Mitigation Framework for Strategic Materials

REPORT DOCUMENTATION PAGE	*Form Approved* *OMB No. 0704-0188*

1. REPORT DATE (DD-MM-YY)	2. REPORT TYPE	3. DATES COVERED (From – To)
April 2013	Final	

4. TITLE AND SUBTITLE Strategic Material Shortfall Risk Mitigation Optimization Model (OPTIM-SM)	5a. CONTRACT NO. DASW01-04-C-0003
	5b. GRANT NO.
	5c. PROGRAM ELEMENT NO(S).
6. AUTHOR(S) James S. Thomason (Project Leader), D. Sean Barnett , James P. Bell , Jerome Bracken, Eleanor L. Schwartz	5d. PROJECT NO.
	5e. TASK NO. DE-6-3247
	5f. WORK UNIT NO.
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Institute for Defense Analyses 4850 Mark Center Drive Alexandria, VA 22311-1882	8. PERFORMING ORGANIZATION REPORT NO. IDA Document D-4811
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) Defense Logistics Agency (DLA) - Strategic Materials 8725 John J. Kingman Road Fort Belvoir, VA 22060	10. SPONSOR'S / MONITOR'S ACRONYM(S) DLA
	11. SPONSOR'S / MONITOR'S REPORT NO(S).

12. DISTRIBUTION / AVAILABILITY STATEMENT

Approved for public release; distribution is unlimited.

13. SUPPLEMENTARY NOTES

14. ABSTRACT

The Department of Defense periodically assesses the potential for shortfalls of strategic and critical materials to occur during crises or military conflicts and recommends to Congress strategies for mitigating those shortfalls. This document presents a model the Institute for Defense Analyses has built and used to identify strategies that would minimize the expected total risk from shortfalls while satisfying constraints on expected total cost and expected risks arising from shortfalls in individual materials. DoD's analytical process selects materials of interest, estimates the material shortfalls occurring in one or more planning scenarios, assesses shortfall risk, identifies promising shortfall mitigation strategies, and assesses the strategies' relative costs and mitigation effectiveness. This model uses those results and identifies an optimal set of strategies for mitigating the shortfalls, within cost and risk constraints set by the user. It does so by solving a linear or non-linear programming problem (depending on the form of the risk function—the assumed relationship between shortfall size and shortfall consequences). This document demonstrates the functionality of the model by presenting analyses performed on data developed for the DOD *Strategic and Critical Materials 2013 Report on Stockpile Requirements*.

15. SUBJECT TERMS

strategic, critical, material, stockpile, shortage, shortfall, risk, cost, assessment, management, strategy, mitigation, linear program, model, optimize, Department of Defense, Defense Logistics Agency, Institute for Defense Analyses

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NO. OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT	b. ABSTRACT	c. THIS PAGE	UU	58	19b. TELEPHONE NUMBER (Include Area Code)
U	U	U			